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On the Possibility of Supertasks

Süpergörevlerin Olabilirliği Üzerine

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Abstract: Supertask, a notion in analytic philosophy, is the process of performing infinite number of operations in a finite amount of time. In this article, we argue about the logical possibility of performing a supertask with a focus on the analysis of S.C. Chihara's discussion paper, and we try to find answers to his questions from a mathematical point of view. In the later sections of the paper, we give some known supertask models in philosophy and mathematics.

Keywords: Supertask, infinite, transfinite, logic, computation.

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Supertasks refer to infinite number of operations executed in a finite amount of time. It has been a puzzling subject for philosophers and philosophical minded mathematicians. Since any kind of infinitary computation will naturally involve procedures having infinitely many computational steps, i.e. supertask procedures, the supertask concept seems to be central to any understanding of the nature of infinitary computability. However, some supertask procedures seem more plausible than the others. Although it is not easy to make a distinction between logical and physical impossibility, we will discuss that some supertask models are more "logically" possible, and some may be even "physically" possible as well. We are not claiming that supertasks are completely logical and possible, but arguing that to some extent some of the models seem to be more possible to have than the others.

Zeno of Elea (450 B.C.) was perhaps the first who dealt with the supertask concept and argued that it is impossible to go from here to there. To go to your destination, you must first get the halfway, and to get the halfway you must first get halfway to the halfway point, and so on. Hence one is required to perform infinitely many tasks to move from here to somewhere else. Since one cannot perform infinitely many tasks, Zeno claimed that motion is impossible.

A more recent supertask was proposed by the philosopher J.F. Thomson (1954: 1). Imagine a mechanism which contains a special kind of lamp and which goes on for ½ minute, off for ¼ minute, on for ⅛ minute, and so on. After exactly 1 minute, is the lamp on or off? The literature is of full of answers. One can convert this problem to the problem of finding the value of Grandi's series. It is expressed by the series

$$\sum_{n=0}^{\infty}(-1)^n$$

Now the sum of this series could be 1 or 0 depending on how you parenthesize the expressions in the summation. Therefore, Grandi's series is known to be divergent. Another example is the super-π machine, which writes out the successive digits of π on a tape. Imagine the machine writes out the first digit in ½ minute, the next digit in ¼ minute, and so on. Hence, all the digits will be written out at the end of exactly 1 minute. By this way, infinite number of tasks is performed in a finite amount of time. However, because there is no last step in such a process, Chihara doubts the possibility of the completion of such a procedure (1965: 80). We will focus on Chihara's objection in his paper here and stand on the positive side to find an answer. The objection is that such procedures do not have the last step. Since it does not have a last step it will not be an algorithm.[1] We then have to make a choice between either not concerning about the possibility of completing an infinite task, or we allow a model to complete infinitely many steps in a finite amount of time. We follow the second choice and argue that some supertask models are more suitable for the second scheme. We make a remark here that some philosophers such as Russell and Whitehead argued that Zeno's paradox would be solved if the series

$$\sum_{n>0} \frac{1}{2^n}$$

has the value 1 since this shows that the distance to be travelled by the the traveller and the amount of time to reach the destination must be finite even though the distance is finite but the distance can be divided into an infinite number of sections (Chihara, 1965: 76). This was also said by Hermann Weyl and Frank Wilczek. They say:

> If the segment of 1 really consists of infinitely many subsegments of length ½, ¼, ⅛, etc., as chopped-off wholes, then it is incompatible with the character of the infinite as incompletable that Achilles should have been able to traverse it all. If one admits this possibility, there is no reason why a machine should not be capable of completing an infinite sequence of distinct acts of decision within a finite amount of time; say, by supplying the first result after ½ minute, the second after another ¼ minute, the third ⅛ minute later than the second, etc. In this way it would be possible, provided the receptive power of the brain would function similarly, to achieve a traversal of all the natural numbers and thereby a 'yes' or 'no' decision regarding any existential question about natural numbers! (Weyl & Wilczek, 2009: 42).

We now turn to Max Black's analysis on the structure of space. He says:

[1] By algorithm, in a classical sense, we mean a well defined finite procedure such that that at any step of the computation, we uniquely determine what happens in the next step.

> If a line in space actually consists of infinitely many points, no motion at all is possible, for the smallest shift of position would involve the crossing of infinitely many points, i.e. the actual performance of an infinite number of acts (Black, 1959: 90).

We have to agree with Black's claim. If the space were continuous, it would be dense. Hence for any two points through a line in space, there would be a point strictly between them. To move from here to somewhere, either the space should be discrete or an infinite amount of time should elapse if we are moving in a continuous space. Zeno of Elea must have assumed that the space is continuous. Even if we assume that the space is continuous, "a physical property applied on space" admits discreteness. Here the physical property that is applied might the motion, or it might be the physical object itself which is bounded by finiteness.

If we again consider super-π machines, J.F. Thomson argues the logical impossibility of this machine (1959: 5). He claims that if such a machine is possible, then so is one that indicates on a dial whether the first machine has printed an odd digit or an even digit. Then what does the second machine indicate when the first machine completes its whole task? He continues saying that if either odd or even is indicated, the first machine must just have printed a digit and hence could not be finished. But either odd or even must be indicated on the dial. So Thomson derives a contradiction here. Paul Benacerraf criticizes this move (1962: 770). He points out that, as Chihara quotes in his paper:

> Thomson does not show that the second machine must indicate odd or even when the first machine finishes its computation. Indeed his question that - What does the second machine indicate when the first completes it task?- is unanswerable. But one cannot conclude that it is unanswerable because there is a contradiction in the notion of such a super-machine, since it seems quite reasonable to maintain that not enough information about the machines was supplied to answer it (Chihara, 1965: 80).

Similarly, for Thomson's lamp example, Benacerraf claims that nothing follows from the states of the lamp inside the series about the state of the lamp after the series. This criticism is widely accepted. V.C. Müller, in his recent work, also agrees with Benacerraf and Chihara for objecting Thomson. He calls the logical gap between the case inside the infinite series and the case after the series, the *Benacerraf gap*. He says:

> I propose that the defender of hypercomputing has to bridge the Benacerraf gap, in order to generate an output. It is crucial for the understanding of the Benacerraf gap to keep in mind that there is no such thing as 'the last state' or the 'the last step' in the series, and accordingly, no last step that can determine the state of the lamp (Müller, 2011: 88).

Müller's concern seems similar to Chihara's that the completion of the last step is problematic. Since this seems like a mathematical problem of defining an action to be taken at a transfinite stage, we will turn back to this issue when we introduce infinite time Turing machines.

G.E.L. Owen claims that it is beside the point to maintain as a general solution of the Zeno's paradox "that an infinite division can be completed" (1957: 204). Now let I_n denote the closed interval with the end points $1-(1/2)^{n-1}$ and $1-(1/2)^n$. Take the Achilles and the tortoise example for the time being. He says:

> For the sequence of moves which Achilles must take to catch the tortoise correspond to the sequence of end points of the intervals I_n. Now imagine Achilles marking in some way the end of each stage of his journey in which he arrives at a point reached by the tortoise in the previous stage (Chihara, 1965: 81).

He claims that Owen interprets Zeno to be arguing that if Achilles is thought to have finished this task, we should be able to ask about the position of the last two marks: If they are in the same place there is no stage determined by them, and if there is any small distance between then this distance is the smallest stage in an infinite set of diminishing stages and therefore the course is infinitely long and not just infinitely divisible.

Zeno's dichotomy paradox is also puzzling. If Achilles has to traverse the interval (0, 1), he must first traverse the interval (0, ½). Then he must first travel the interval (0, ¼), and so on ad infinitum. Thus it turns out that we can expect Achilles to do something analogous to counting all numbers in reverse order, i.e. counting $n+1$ before n. Here the problem is not to find how we completed this task but how we started. Of course the problem of how we started is essentially the same as the problem of how we completed the task when we had to do it in the forward order. The type of problem uncovered by Zeno is not restricted to motion in space. We can find similar difficulties by asking similar questions in different forms. Another example is the bouncing ball problem. The question is that will a bouncing ball ever come to rest? The physics of a bouncing ball can be mathematically analyzed in such a way, ignoring factors other than rebound, to predict an infinite number of bounces.

Now if we want to seek for a mathematical solution to completing a supertask we need to remind the reader what Chihara was concerning about the infinite machines, such as super-π machines, completing their tasks. He quotes:

> The difficulty, as I see it, is not insufficiency of time, tape, ink, speed, strength of material, power, and the like, but rather the inconceivability of how the machine could actually finish its super task (Chihara, 1965: 80).

He argues that when the machine completes its task and shuts itself off, we should be able to look at the tape to see what digit was printed last, but if the machine finishes printing all the digits which constitute the decimal expansion of π , no digit can be the last digit printed. So the problem is that whether or not we can really conceive of such a machine actually completing its computation.

We believe Chihara's concern about the completion of supertasks is a matter of mathematical definition for the corresponding hypercomputing model. If Chihara suspects that the last step of computation is not conceivable, then one may explicitly define the *limit* behaviour of such models of supertask computation. For example, J.D. Hamkins and A. Lewis in propose a supertask model called *infinite time Turing machine* (2000: 1).

A *classical Turing machine* is a mathematical model of computation, considered to be a "complete" model in a sense that it captures the notion of "algorithmic" computation, which contains an infinite tape divided into cells with a tape head to read/write symbols on the tape cell, a tape alphabet, a finite set of states, and a finite set of instructions. The computation starts by reading the leftmost symbol of the input written on the tape, then following the instructions, we move the tape head, write a symbol on the cell, change the state of the machine if necessary. The *configuration* of the machine at any stage is determined by the position of the tape head, the content of the tape, and the state of the machine. This model is believed to capture the notion of algorithmic computability and this is called the *Church-Turing thesis*. The statement is not mathematical but rather philosophical because of that "algorithmic computability" is a well-defined mathematical term, where a Turing machine is a mathematical object. An infinite time Turing machine is just like a classical 3-tape Turing machine which contains three infinite tapes (input tape, scratch tape, output tape) divided into cells, a tape head to read/write symbols on the tape cell, a tape alphabet, a finite set of states, and a finite set of instructions. The computation is carried out the same way as in the classical Turing machine.

However, they allow this machine to compute infinitely many steps. Although it is not clear that whether it is the time or the computation step that they extend to infinity, since we cannot really talk about the notion of time in pure mathematics, what is new is the transfinite behaviour of this machine. Given any stage α, if we know the configuration of the machine at that stage, we can uniquely determine the configuration at next stage. The authors define the transfinite action of the machine as follows: at any limit ordinal stage[2], the head resets to the left most cell and the machine is placed in a special distinguished *limit* state. We then take the limit of the cell values on the tape as follows: If the values appearing in a cell have converged before the limit stage, then the cell gets the limiting value at the limit stage. Otherwise, in the case that the cell values have alternated from 0 to 1 and back again unboundedly often, we make the limit cell value 1. In other words, we take the *limit supremum* of the cell values. This describes the configuration of the machine at any limit ordinal stage β and the machine can go on computing to $\beta+1$, $\beta+2$ and so on, eventually taking another limit at $\beta+\omega$ and so forth. If the machine reaches the *halt* state at any stage, the computation ends and the output is whatever is written on the output tape.

[2] An ordinal number β is called a *limit ordinal* if $\beta = \alpha + 1$ for no ordinal α. We call the least transfinite (and limit) ordinal as ω which is the order type of the set of natural numbers $\{0,1,2\ldots\}$. Basically, ω is a transfinite number which is greater than any number in the set of naturals.

Chihara does not worry about the physical properties of the computation, i.e. speed, time etc., but we think it matters. If we take infinite time Turing machines into account, when infinite amount of time (or steps) is allowed, the machine will perform the distinguished limit action.

However, one might ask "Will an infinite time Turing machine ever elapse infinite time to perform its limit action?" One can give a positive answer to this question. In fact, some supertask models have been proposed by M. Hogarth in the context of relativity theory that such models to perform infinite computation do exist (Hogarth, 1992: 173-181). For a more philosophical account of this work, we refer the reader to Earman and Norton (1993: 22-42). Suppose that we want to find an answer to some number theoretic conjecture that can be confirmed with a single numerical example. Roughly speaking, the way to solve the problem is to board a rocket, while setting some colleagues to work on earth looking for an example. While we fly faster and faster around the earth, our colleagues, their students, and the students of them, and so on, continue the exhaustive search, with the agreement that if they ever find an example, they will send a radio signal up to a rocket. Meanwhile, by accelerating sufficiently fast towards the speed of light, but never exceeding, it is possible to arrange that the relativistic time contraction on the rocket will mean that a finite amount of time on the rocket corresponds to an infinite amount of time on the earth. The point is that by means of the communication between the two reference frames, what corresponds to an infinite search can be completed in a finite amount of time.

In a way, Chihara's concern on the completeness of supertasks can be settled by defining a transfinite action in a well-defined mathematical model. This means that, unlike his claim that the amount of time spent on the computation is not important, after sufficiently long time is elapsed, the machine will perform its transfinite action. Therefore one solution to Chihara's problem could be infinite time Turing machines on Malamenth-Hogarth spacetime.

However, it appears that physical possibility and conceptual possibility of supertasks are independent from each other. While Zeno's paradox seems to be puzzling, Malamenth-Hogarth spacetime model, seems to allow supertasks even on a physical basis. The problem of course is that, when studying the conceptual possibility of supertasks, we use the continuity phenomenon in physical theories involving motion and we take for granted that the space as a continuum. This is a problematic issue and there are different interpretations whether or not one should take it as a continuum. If we take it as a continuum then we have paradoxes of infinity. But if we take it as a discrete universe, then what happens when we attempt to measure sufficiently small quantities? These of course should be considered by philosophers or maybe physicists.

Over the centuries Zeno's paradoxes have played an important role in pointing to fundamental questions in logic, philosophy, mathematics, and the physics of motion. The discussion of infinite tasks and paradoxes that they bring is very deep. This is certainly not the last word on supertasks. We should not be surprised if philosophers uncover more logical and conceptual difficulties about supertasks. There are non-trivial supertasks that are not problematic. Transfinite computing machines on Malamenth-Hogart spacetime may be very well taken to be of them.

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Özet: Analitik felsefenin kavramlarından biri olan süpergörev, sonlu bir zaman aralığında sonsuz işlem yapmaya denir. Bu makalede süpergörevlerin mantıksal açıdan uyumluluğu, temelde S.C. Chihara'nın derlemesindeki fikirler analiz edilerek tartışılmış ve sorularına matematiksel açıdan cevaplar aranmıştır. Makalenin devamında ise felsefe ve matematikte bilinen bazı süpergörev modelleri verilmiştir.

Anahtar Kelimeler: Süpergörev, sonsuz, sonluötesi, mantık, hesaplama.