Is the Church-Turing Thesis the new Pythagoreanism?

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Effective procedures (algorithms)

- Roughly defined as a finite, step-by-step, deterministic procedure.
- An early example: Euclid's method for finding the *greatest common divisor* of any given two natural numbers.
- Post-Euclid: Taking effective computability for granted.

Entscheidungsproblem

Hilbert and Ackermann, *Grundzüge der theoretischen Logik (Principles of Mathematical Logic)*, 1928

Entscheidungsproblem: The problem of deciding whether a given sentence in first-order logic is valid or not.[1]

Note that Entscheidungsproblem is solvable for propositional logic.

[1] Hilbert, known to be a formalist, believed in the non-existence of unsolvable problems. Would this make him a truth-value (semantic) realist?

Models of effective computability

- Gödel's *general recursive functions*
- Kleene's μ-*recursive functions*
- Church's λ-*calculus*
- Turing's *Turing machines*

The unsolvability of *Entscheidungsproblem* was shown by Alonzo Church in 1936.[2]

[2] A. Church, *An unsolvable problem of elementary number theory*, The Journal of Symbolic Logic, Vol.1, p.73-74, 1936.

Church-Turing Thesis

All these models of effective computability are shown to be equivalent. However, Turing machines are nowadays accepted as such models which naturally describe the intuitive notion of computability.

Church-Turing Thesis: *Every "effectively" computable function is computable by a Turing machine.*

Variants of the Church-Turing Thesis

Computability, unless used for a specific setting, is a very broad term. When do we count something as computable?

Physical Church-Turing Thesis: Physical computability=Turing computability

Human Church-Turing Thesis: Human computability=Turing computability

Gualtiero Piccinini suggested two versions of the Physical Church-Turing Thesis: *Modest* and *bold* versions.[3]

[3] G. Piccinini, *Computationalism, The Church-Turing Thesis, And The Church-Turing Fallacy*, Synthese, **154**, p.97-120, 2007.

What Church-Turing Thesis is not

The Church-Turing Thesis does not say that there is a limit to all possible computing *systems*.
The Church-Turing Thesis does not say that minds are machines. Because we do not know if the mind works in an effective fashion.
Finally, according to Piccinini, the Church-Turing Thesis does not necessarily imply *computationalism*.[4]

[4] Computationalism is the view that cognitive capacities of the mind have a computational explanation or, more strongly, that cognition is a type of computation.

Pythagoreanism, revisited

Broadly speaking, Pythagoreanism claims that everything in the universe and the so called harmony can be expressed by finite constructions of natural numbers.
Expressive-completeness of natural numbers in Pythagoreanism.

Two basic assumptions of Pythagoreanism:
1. The universe is believed to be a closed system.
2. This system can be reduced to numbers.

A crisis in mathematics

Discovery of that $\sqrt{2}$ is irrational.
Natural numbers are often associated with concepts like *rationality*, *finiteness*, *discreteness*, *simplicity* and *preciseness*.

Now we should either accept irrationality or adopt the view that irrational numbers have nothing to do with the "physical reality" of the universe, hence save the Pythagoreanistic doctrine.
There is however another way to argue for Pythagoreanism.

Church-Turing Thesis as the *new Pythagoreanism*

Church-Turing Thesis (in its general form) implies that intuitive computability, present in the human mind and in physics, can be reduced to Turing mechanics.
Then, according to the Church-Turing Thesis, there is no effective computation which lies beyond the expressive-completeness of Turing mechanics.
Despite that Pythagoreanism was refuted by its own followers, the Church-Turing Thesis has not been refuted yet.

The *new Pythagoreanism* (continued)

Is Turing mechanics (which is clearly representable in the domain of natural numbers) all what it takes to define the intuitive notion of computability?
Then, this thesis tells us that Turing mechanics are sufficient enough to express any intuitive computation as natural numbers have the similar expressive-completeness in Pythagoreanism. Should we accept this expressive-completeness of the Church-Turing Thesis? Or should we look for the $\sqrt{2}$ of effective computation?

The *new Pythagoreanism* (continued)

One may take the Church-Turing Thesis, for these reasons, as a candidate Pythagoreanistic philosophy. However, it is not clear whether that is really the case because the Church-Turing Thesis has not been proved or disproved yet. Relying on the past 80 years, we believe that it is true. Nevertheless, we can refute it either by

(i) Finding a Turing-computable function which does not seem to be *intuitively* computable. (Not likely)
(ii) Finding a *process* and convincing ourselves that it is *computable* but that cannot be simulated by Turing machines. (More likely)

Supertasks

Supertasks, whether one accepts them or not, might be a new paradigm for computability.

Supertasks
Completing an infinite task within a finite interval is called a *supertask*.

Entscheidungsproblem becomes solvable for supertasks.

On the possibility of supertasks

- Logical possibility

Criticized by J.F.Thomson.[5]
Supported by P. Benacerraf.[6]

- Physical (theoretical) possibility

There are spacetime structures which are consistent with general relativity in which performing supertasks is possible.

[5] J. F. Thomson, *Tasks and Super-tasks*, Analysis, **15** (1): 1-13, 1954.

[6] P. Benacerraf, *Tasks, Super-tasks, and Modern Eleatics*, The Journal of Philosophy, **59** (24): 765-784, 1962.

What should we infer?

Question: Is the Church-Turing Thesis the new Pythagoreanism?
Answer: It depends on how we define *computability*. If we restrict ourselves to just Turing-computability, then yes. If not, then we should redefine computability and perhaps include supertasks as legitimate *computable* processes. What then?

Effects on philosophy of mathematics

We think about some foundational notions of mathematics: Numbers/sets (ontology), proofs/models (epistemology), and logic/computation (methodology).
If we have to adopt the Church-Turing Thesis, one should also think about its consequences in philosophy of mathematics. We want to know what might be a *suitable* philosophy of mathematics. Of course several different philosophies can be suggested. But one should particularly aim to answer the following questions.
What is a compatible ontological philosophy of mathematics?
What is a compatible epistemological philosophy of mathematics?
What is a compatible methodology of mathematics?

Metaphysical point of view

Three big ontological philosophies: *Realism, idealism and nominalism.*
Three fundamental properties of realism: *Existence, independence, abstractness.*
Radical realism: Adding *uniqueness* property, i.e. there is one and only one Platonic universe of mathematical objects.
Inconsistencies between terms and objects. One term for one meaning
Quine's *"No entity without identity"* is adopted in a different setting, where the term *identity* is now used to refer identification, i.e. naming.
CTT seems to be incompatible with radical realism. That is not to say that we should accept multiverses of mathematical objects. Due to other epistemological problems in multiverse theory, we might have to be more careful about choosing a realist philosophy for the Church-Turing Thesis.

Idealist abstractionism

We cannot discard the intuitionistic feature of the Church-Turing Thesis since the thesis statement contains unclear notions like *computable*, *effective* and etc. Yet it also has a formalistic nature due to that intuitive computability is associated with a formal mathematical object, i.e. Turing machine model.
Nominalism cannot be an option. If Turing machines (or equivalent physical/mathematical models) did not exist, then we would have nothing to compute with. We shall perhaps consider idealism as an ontological philosophy along with the idea of formal machinery *abstraction*. How we define *idealist abstractionism* is another topic that should be discussed another time.

Thank you