

**MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF MATHEMATICS**

**MATH 493 Philosophy of Mathematics
Course Syllabus
Fall 2023**

Instructor: Assoc. Prof. Dr. Ahmet Çevik

Office hours: By appointment

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Prerequisite course: None. However, basic knowledge in logic and set theory is desired.

Course objective:

The main goal of this course is to give students an overall idea about various philosophical positions of mathematics and to study selected foundational subjects such as computability, Gödel's incompleteness theorems, and the Axiom of Choice. By taking this course, students will be able to develop skills in

- i. understanding and classifying problems in the philosophy of mathematics,
- ii. engaging in a philosophical discussion concerning mathematical disciplines,
- iii. understanding the nature of mathematics and the effects of different philosophical positions on mathematical practice,

Course content:

1. Introduction

Subject matter of philosophy of mathematics, basic terminology, questions in the philosophy of mathematics, mathematical preliminaries.

2. Mathematical Platonism (Realism)

Ontological and epistemological realism, Plato vs. Aristotle on mathematics.

3. Intuitionism

Kant's philosophy of mathematics, Brouwer's constructivism, impredicativity.

4. Logicism

Frege, Russell, Carnap and logical positivism.

5. Formalism

Term formalism vs. game formalism, deductivism, Hilbert's programme.

6. Computability and Incompleteness

Turing Machines, Church-Turing Thesis, Gödel's Incompleteness Theorems.

7. Philosophy of the Infinite

Infinity in ancient Greece, Zeno's paradoxes, Nicholas of Cusa, Cantor's theory of sets, Continuum Hypothesis.

8. Axiom of Choice

Definition of the Axiom of Choice (AC), equivalents of AC, paradoxical consequences, arguments concerning the 'plausibility' or otherwise.

9. Mathematical naturalism (time permitting)

Gödel's realism, Quine, Maddy and set-theoretical realism, Maddy's naturalism.

Assessment:

Midterm I (%30)

Midterm II (%30)

Final (%40)

Reference books:

1. A. Çevik, *Philosophy of Mathematics: Classic and Contemporary Studies*, CRC Press/Taylor & Francis, 2021.
2. P. Benacerraf and H. Putnam, *Philosophy of Mathematics: Selected readings*, Cambridge University Press, 1983.
3. J. D. Hamkins, *Lectures on the Philosophy of Mathematics*, MIT Press, 2020.
4. M. Colyvan, *An Introduction to the Philosophy of Mathematics*, Cambridge University Press, 2012.
5. S. Shapiro, *Thinking about mathematics: Philosophy of Mathematics*, Oxford University Press, 2000.
6. O. Linnebo, *Philosophy of Mathematics*, Princeton University Press, 2017.
7. S. B. Cooper, *Computability Theory*, CRC Press, 2003.
8. R. I. Soare, *Turing Computability*, Springer, 2016.
9. G. Boolos and R. C. Jeffrey: *Computability and Logic*, Cambridge University Press, 2007.