MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF MATHEMATICS

MATH 432
COMPUTABILITY THEORY

Course syllabus

Lecturer: Assoc. Prof. Ahmet Çevik
Lecture day & hours: Wednesday, 14:40-17:30.
Office hours: By appointment (e-mail: acevik@metu.edu.tr). Office: Z-41
Course credit: (3-0)3
Course category and level: Elective undergraduate (Advanced level)
Prerequisites: MATH 111 is essential. Some familiarity with mathematical logic is desired.

Catalogue description: Origins of computability, Gödel's incompleteness theorems, partial recursive functions, Turing machines, Church-Turing Thesis, decidability, recursion theorem, s-m-n theorem, recursively enumerable (r.e.) sets, computable approximations, halting problem, creative sets, simple sets, Turing reducibility, Turing degrees, properties of Turing degrees, recursively enumerable degrees, joins, Turing jump, arithmetical hierarchy, limit lemma, finite extension method, co-infinite extension method, jump classes, jump inversion, low and high degrees, finite injury priority method, r.e. permitting, computable domination, minimal degrees.

Course objective: There are four pillars of mathematical logic: Proof theory, model theory, set theory, and computability theory (also known as *recursion theory*). Computability theory is the mathematical study of unsolvability. It studies the limits of effectively computability and characterizes the incomputability of subsets of natural numbers. By taking this course, students will be able to understand the fundamental theorems and notions in computability theory such as Gödel's Incompleteness Theorems, relative computability, Turing degrees and their relations, construction methods including finite injury priority method and finite extension method, and methods for defining recursively enumerable sets and degrees; they will be able to apply the construction methods in other branches of mathematics; compare the theorems in other areas of mathematical logic with recursion theoretic results in order to devise a method for solving problems in similar domains; develop innovative construction methods for defining effectively enumerable sets in game-like settings.

Course schedule (tentative):

Week	Topic
Week 1	**Origins and formalism:** Origins of computability, Hilbert's programme, Language of arithmetic, arithmetization of syntax.
Week 2	**Primitive recursion and incompleteness-I:** Gödel's Incompleteness Theorem.
Week 3	**Primitive recursion and incompleteness-II:** Gödel's Incompleteness Theorem (continued), consequences of incompleteness.
Week 4	**Turing machines:** Partial recursive functions, μ-recursive functions, Turing machines.
Week 5	**Decidability:** Universal Turing machines, Church-Turing Thesis, decidable sets.
Week 6	**Fundamental theorems of recursion theory:** Recursion Theorem, s-m-n theorem, padding lemma, computable approximation, recursively enumerable sets.
Week 7	**Undecidable sets:** Halting problem, creative sets, simple sets, permitting method.
	MIDTERM I
Week 8	**Relative computability:** Oracle Turing machines, Turing reducibility, Turing degrees.
Week 9	**Recursively enumerable degrees and joins:** Use principle, recursively enumerable degrees, joins, the jump operator.
Week 10	**Arithmetical Hierarchy:** Complexity of formulas, Post's Theorem, Limit Lemma, Δ_2-approximations.
Week 11	**Finite extension and the co-infinite extension method.** Defining sets with finite extension method, existence of incomparable degrees below 0', Turing degrees form an exact semi-lattice.
	MIDTERM II

	Week 12	**Minimal degrees:** Splitting trees, minimal degree construction below 0''. Stating the existence of minimal degrees below 0'. Stating Sack's Density Theorem.	
	Week 13	**Jump classes:** Low and high degrees, jump inversion, Finite injury priority method and the Frieberg-Muchnik Theorem, computable domination, hyperimmune-free degrees.	
	Week 14	Selected topics (Effectively closed sets, basis and anti-basis theorems)	

Assessment:
Midterm I (%30)
Midterm II (%30)
Final (%40)

Reference books:

1. R. I. Soare, *Turing Computability*, Springer, 2016. (Advanced undergrad level)
2. S. B. Cooper, *Computability Theory*, Chapman & Hall /CRC, 2004. (Advanced Undergrad level)
3. R. I. Soare, *Recursively Enumerable Sets and Degrees*, Springer, 1987. (Graduate level)
4. R. G. Downey, D. R. Hirschfeldt, *Algorithmic Randomness and Complexity*, Springer, 2010. (Graduate level)
5. A. Nies, *Computability and Randomness*, Oxford University Press, 2009. (Graduate level)
6. M. Davis, *Computability and Unsolvability*, Dover Publications, 1982. (Advanced Undergrad level)
7. N. J. Cutland, *Computability: An Introduction to Recursive Function Theory*, Cambridge University Press, 1980. (Advanced Undergrad level)