MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF MATHEMATICS

MATH 493 Philosophy of Mathematics
Course Syllabus
Spring 2025

Instructor: Assoc. Prof. Dr. Ahmet Çevik
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Office hours: By appointment
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Prerequisite course: Formally there is no prerequisite course. However, some knowledge in logic and set theory (e.g., ordinal and cardinal numbers) is desired. Familiarity with mathematical logic is bonus. Solid understanding of MATH111 is essential.

Course objective:

The main goal of this course is to give students an overall idea about various philosophical positions of mathematics and to study selected foundational subjects such as computability, Gödel's incompleteness theorems, and the Axiom of Choice. By taking this course, students will be able to develop skills in

  i.   understanding and classifying problems in the philosophy of mathematics,
  ii.   engaging in a philosophical discussion about the nature of mathematics,
  iii.   understanding the effects of different philosophical positions on mathematical practice.

Course content:

1. **Introduction**
   Subject matter of philosophy of mathematics, basic terminology, questions in the philosophy of mathematics, mathematical preliminaries.
2. **Mathematical Platonism (Realism)**
   Ontological and epistemological realism, Plato vs. Aristotle on mathematics.
3. **Intuitionism**
   Kant's philosophy of mathematics, Brouwer's constructivism, impredicativity.
4. **Logicism**
   Frege's logicism, Russell's paradox, Carnap and logical positivism.
5. **Formalism**
   Term formalism vs. game formalism, deductivism, Hilbert's formalisation programme.
6. **Incompleteness and Computability**
   Gödel's Incompleteness Theorems, Turing Machines, Church-Turing Thesis.
7. **Philosophy of the Infinite**
   Infinity in ancient Greece, Zeno's paradoxes, Nicholas of Cusa, Cantor's theory of sets, Continuum Hypothesis.
8. **Axiom of Choice**
   Definition of the Axiom of Choice (AC), equivalents of AC, paradoxical consequences, arguments concerning the 'plausibility' or otherwise.

9. **Mathematical naturalism (time permitting)**
   Gödel's realism, Quine, Maddy and set-theoretical realism, Maddy's naturalism.

Assessment:
Midterm I (%30)
Midterm II (%30)
Final (%40)

Reference books:

1. A. Çevik, Philosophy of Mathematics: Classic and Contemporary Studies, CRC Press/Taylor & Francis, 2021.
2. P. Benacerraf and H. Putnam, Philosophy of Mathematics: Selected readings, Cambridge University Press, 1983.
3. J. D. Hamkins, Lectures on the Philosophy of Mathematics, MIT Press, 2020.
4. M. Colyvan, An Introduction to the Philosophy of Mathematics, Cambridge University Press, 2012.
5. S. Shapiro, Thinking about mathematics: Philosophy of Mathematics, Oxford University Press, 2000.
6. O. Linnebo, Philosophy of Mathematics, Princeton University Press, 2017.
7. S. B. Cooper, Computability Theory, CRC Press, 2003.
8. R. I. Soare, Turing Computability, Springer, 2016.
9. G. Boolos and R. C. Jeffrey: Computability and Logic, Cambridge University Press, 2007.