MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF MATHEMATICS

MATH 406 INTRODUCTION TO MATHEMATICAL LOGIC AND MODEL THEORY

Course Syllabus

Lecturer: Ahmet Çevik
Lecture day & hours: Wednesday, 14:40-17:30
Office hours: By appointment (e-mail: acevik@metu.edu.tr)
Course credit: (3-0)3
Course category and level: Elective undergraduate (4^{th} and 3^{rd} year).
Prerequisites: None. However, solid understanding of MATH111 is necessary.

Catalogue description: Propositional logic, predicate logic, structures, embeddings, models, compactness, completeness, Löwenheim-Skolem Theorem, model completeness, Vaught's test, types, atomic models, universal models, saturated models, primitive recursive functions, Turing machines, Gödel numbering, recursively enumerable sets, Gödel's Incompleteness Theorem.

Course objective: Mathematical logic consists of four branches: Proof theory, set theory, model theory, recursion theory (computability theory). The main goal of this course is to teach students some fundamental results in mathematical logic with primary focus on model theory. If time permits, some basic topics in computability will be covered, such as primitive recursive functions, recursive enumerability, Gödel numbering of formulas, and Gödel's Incompleteness Theorem.

By taking this course, students will be able to

i. explain the subject matter of the main fields of mathematical logic,
ii. prove basic theorems of mathematical logic,
iii. use and apply first-order predicate logic,
iv. apply model theoretic and recursion theoretic techniques,
v. understand the limits of formal reasoning,
vi. describe the problems of meta-mathematics.

Course plan:

I. Propositional Logic and First-Order Predicate Logic
Week 1: Propositions, connectives, truth assignments, validity, satisfiability, contradiction, tautology.
Week 2: Consequence and equivalence, rules for derivations, proof by induction on formulas.
Week 3: Compactness and completeness of propositional logic.
Week 4: Language of first-order logic, formulas, sentences, semantics of first-order logic.

II. Structures and Models
Week 5: Structures, size of models, isomorphism, embeddings, elementary equivalence.

Week 6: Examples of structures, elementary substructures and elementary embeddings.
Week 7: *Midterm I*
Week 8: Theories and models, compactness and completeness of first-order predicate logic, Löwenheim-Skolem Theorem.
Week 9: Craig's Interpolation Theorem, decidable theories, recursively axiomatizable theories.

III. Selected Topics in Model Theory

Week 10: Categoricity, model completeness, Vaught's test.
Week 11: *Midterm II*
Week 12: Types, atomic models, universal models, saturated models.

IV. Recursion Theory (time permitting)

Week 13: Primitive recursive functions, Turing machines, Gödel numbering, recursively enumerable sets, recursion theorem, halting problem.
Week 14: Gödel's Incompleteness Theorem.

Assessment:

Midterm I (%30)
Midterm II (%30)
Final (%40)

Reference books:

1. S. Hedman, *A First Course in Logic: An Introduction to Model Theory, Proof Theory, Computability and Complexity*, Oxford University Press, New York, 2004.
2. S. C. Kleene, *Mathematical Logic*, Dover Publications, 1967.
3. E. Mendelson, *Introduction to Mathematical Logic*, CRC Press, 2010.
4. D. Marker, *Model Theory: An Introduction*, Springer-Verlag, New York, 2002.
5. C. C. Keisler and H. J. Chung, Model Theory, North Holland, 1973.
6. M. Davis, *Computability and Unsolvability*, Dover Publications, New York, 1985.
7. S. B. Cooper, Computability Theory, CRC Press, 2003.
8. R. I. Soare, Turing Computability, Springer-Verlag, 2017.